Fill in the following identities. **SCORE: _____ / 14 PTS**

[a] PYTHAGOREAN IDENTITY:

$\cot^2 x = \csc^2 x - 1$

[b] NEGATIVE ANGLE IDENTITY:

$\sec(-x) = \sec x$

[c] SUM OF ANGLES IDENTITY:

$\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

[d] DIFFERENCE OF ANGLES IDENTITY:

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

[e] HALF ANGLE IDENTITY:

$\cos\frac{1}{2}x = \pm\sqrt{\dfrac{1+\cos x}{2}}$

[f] POWER REDUCING IDENTITY:

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

[g] DOUBLE ANGLE IDENTITY: $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

WRITE ALL 3 VERSIONS

If $\cos x = -\frac{3}{5}$ and $\pi < x < \frac{3\pi}{2}$, find the values of the following expressions. **SCORE: _____ / 30 PTS**

Write each final answer as a single fraction in simplest form, including rationalizing the denominator.

[a] $\sin 2x$ $= 2\sin x \cos x$

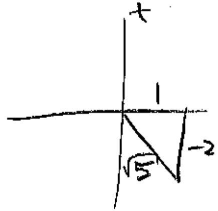
$= 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)$

$= \frac{24}{25}$

[b] $\cos(\arctan(-2) - x)$ $= \cos(t - x)$

$t = \arctan(-2)$

$\tan t = -2$ AND $t \in Q_4$



$= \cos t \cos x - \sin t \sin x$

$= \frac{1}{\sqrt{5}} \cdot \frac{-3}{5} + \frac{-2}{\sqrt{5}} \cdot \frac{-4}{5}$

$= \frac{5}{5\sqrt{5}}$

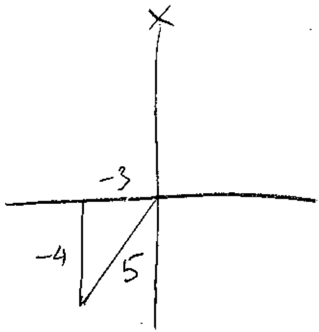
$= \frac{\sqrt{5}}{5}$

[c] $\tan \frac{1}{2}x$ $= \frac{\sin x}{1 + \cos x}$

$= \frac{-\frac{4}{5}}{1 - \frac{3}{5}}$

$= \frac{-\frac{4}{5}}{\frac{2}{5}}$

$= -2$



Prove the identity $\dfrac{(\csc B - \cot B)^2 + 1}{\sec B \csc B - \cot B \sec B} = 2\cot B$.

SCORE: _____ / 14 PTS

$$= \frac{\csc^2 B - 2\csc B \cot B + \cot^2 B + 1}{\sec B\,(\csc B - \cot B)}$$

$$= \frac{2\csc^2 B - 2\csc B \cot B}{\sec B\,(\csc B - \cot B)}$$

$$= \frac{2\csc B\,\cancel{(\csc B - \cot B)}}{\sec B\,\cancel{(\csc B - \cot B)}}$$

$$= 2 \cdot \frac{1}{\sin B} \cdot \cos B = 2\cot B$$

Rewrite $\sin^4 x$ using only the first powers of cosine (and constants and the 4 basic arithmetic operations). **SCORE: _____ / 14 PTS**
Simplify your final answer, which must **NOT** be in factored form, and must **NOT** involve any other trigonometric functions.

$$\sin^4 x = \left(\tfrac{1}{2}(1-\cos 2x)\right)^2$$

$$= \tfrac{1}{4}(1 - 2\cos 2x + \cos^2 2x)$$

$$= \tfrac{1}{4}\left(1 - 2\cos 2x + \tfrac{1}{2}(1+\cos 4x)\right)$$

$$= \tfrac{1}{8}(3 - 4\cos 2x + \cos 4x)$$

Solve the equation $7-4\sin 3x = 6(1-\sin 3x)$. **SCORE: _____ / 14 PTS**

$7-4\sin 3x = 6-6\sin 3x$

$2\sin 3x = -1$

$\sin 3x = -\frac{1}{2}$

$3x = \frac{5\pi}{6}+2n\pi,\ \frac{7\pi}{6}+2n\pi$

$x = \frac{5\pi}{18}+\frac{2n\pi}{3},\ \frac{7\pi}{18}+\frac{2n\pi}{3}$

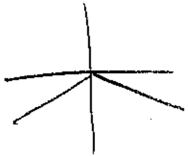
Solve the equation $2\cos 2x + 7\sin x = 0$ algebraically. **SCORE: _____ / 14 PTS**

$$2(1-2\sin^2 x) + 7\sin x = 0$$

$$2 + 7\sin x - 4\sin^2 x = 0$$

$$(2-\sin x)(1+4\sin x) = 0$$

$$\sin x = \cancel{2} \text{ OR } \sin x = -\tfrac{1}{4} \longrightarrow x \in Q_3, Q_4$$



$$x = \pi + \sin^{-1}\tfrac{1}{4} + 2n\pi$$

$$\text{OR } 2\pi - \sin^{-1}\tfrac{1}{4} + 2n\pi$$

$$= 3.3943 + 2n\pi \text{ OR } 6.0305 + 2n\pi$$